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Optical Bistability and Self-Pulsation with Long-Range Hybrid Plasmonic Disk Resonators

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Presentation outline

- **Nonlinear Long-Range Hybrid Plasmonic Travelling-Wave Disk Resonator**
  - Long-Range Hybrid Plasmonic Waveguide
  - Physical System: Side-Coupled Disk Resonator
  - Perturbation Theory & CMT Framework: Kerr effect and Two Photon Absorption
  - System Design
  - Perturbation Theory & CMT Framework: Free Carrier Effects
  - CW Performance Assessment
  - Temporal Response
  - Stability Analysis and Self-Pulsation

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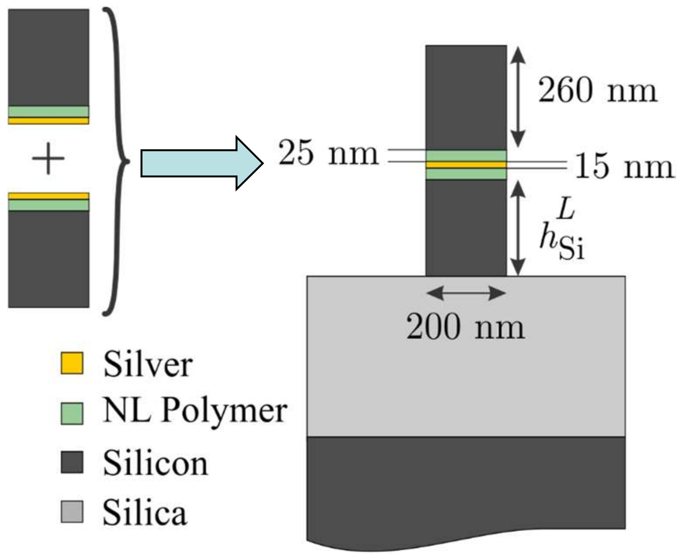
Nonlinear long-range hybrid plasmonic waveguide (NL-LRHPW)

- "Combination" of two CGS with nonlinear polymer **DDMEBT** (n_2 = 1.7×10^{-17} m^2/W)
  - **Silver** for lower resistive losses

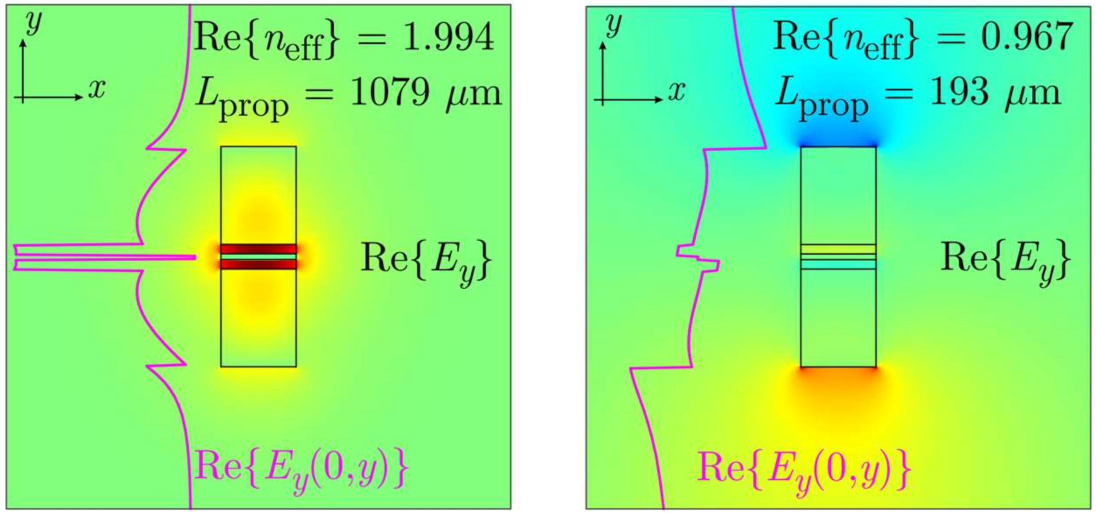


Waveguide Structure
[*Binfeng, JOSA B 26, 2009*]
[*Bian, Opt. Expr. 17, 2009*]

Material Properties
[*Esembeson, Adv. Mater. 20, 2008*]
[*Koos, Nat. Photon. 3, 2009*]
[*Johnson, PRB 6, 1972*]

Two supported modes

Symmetric | Antisymmetric



Symmetric:
- ☑ Low resistive losses
- ☑ Large L_{prop}
- ☑ Strong confinement

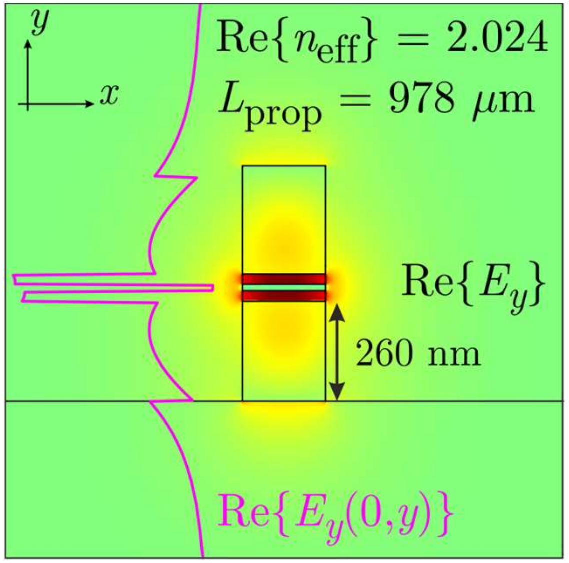
Antisymmetric:
- ☒ High resistive losses
- ☒ Poor L_{prop}
- ☒ Weak confinement

Nonlinear long-range hybrid plasmonic waveguide (NL-LRHPW)

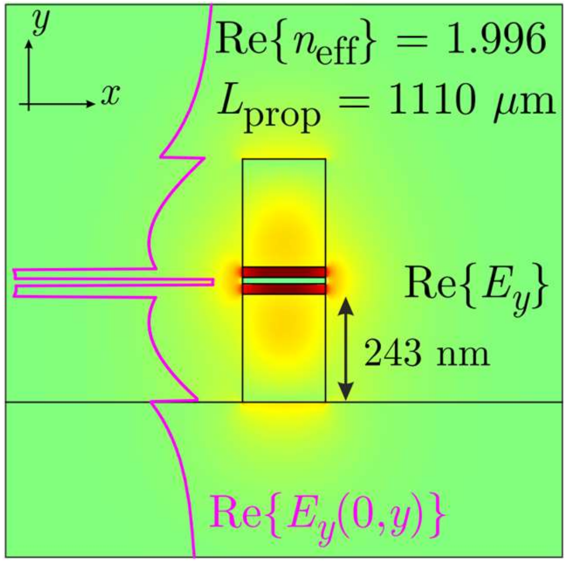
- ❑ Substrate de-symmetrizes LRHPW profile → Higher resistive losses
- ❑ Profile re-symmetrization by h^{L}_{Si} modification → Resistive losses restoration

[*Ma, JOSA B 14, 2014*]

Asymmetric profile



Re-symmetrized profile



- ☑ Large propagation length: $\boldsymbol{L_{prop}}$ **~ 1100 μm** (15 times higher than CGS)
- ☑ Strong confinement: $\boldsymbol{A_{eff}}$ **~ 0.07 μm²** (only 40% larger than CGS)
- ☑ High nonlinear coefficient: $\boldsymbol{\gamma_{wg} \sim 1485 - j11.4\ \ W^{-1}m^{-1}}$
  - Real part: Kerr effect (97% DDMEBT, 3% Si)
  - Imaginary part: TPA in Si → leads to FCEs

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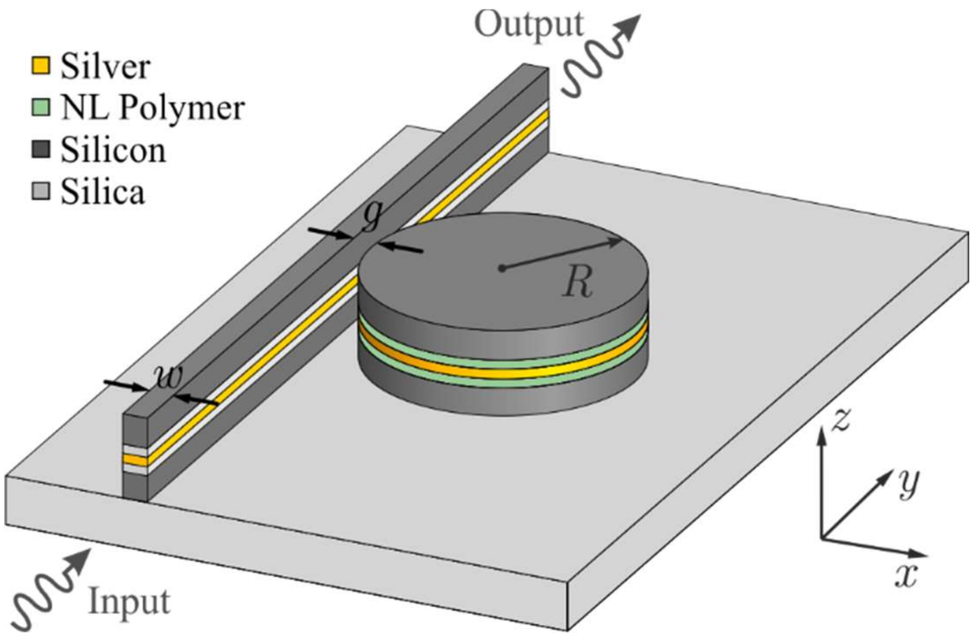
Nonlinear disk resonator structure

Nonlinear disk side-coupled with LRHPW bus waveguide

- ❑ **Intensity build-up** in resonator → Nonlinearity enhancement
- ❑ Disk: Reduced radiation losses → Higher Q
- ❑ Compact structure



[*Tsilipakos, Christopoulos and Kriezis, JLT 34, 1333, 2016*]

$\mathrm{Re}\{\chi^{(3)}\} \rightarrow \Delta\varepsilon'_r \propto |E|^2$ ➡ ω_{res} shifts! ($\Delta\omega<0$ for $n_2>0$)

$\mathrm{Im}\{\chi^{(3)}\} \rightarrow \Delta\varepsilon''_r \propto |E|^2$ ➡ Nonlinear losses (TPA)

BUT

TPA @ Si produces free carriers → FCEs

Free Carrier Dispersion (FCD) $\Delta\varepsilon'_r \propto |E|^4$	Free Carrier Absorption (FCA) $\Delta\varepsilon''_r \propto |E|^4$
⬇	⬇
ω_{res} shifts again! $\Delta\omega>0$	More nonlinear losses

COUNTERACTION

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Modelling Framework (basic idea)

Mathematical Framework

Perturbation Theory
+
Temporal CMT
+
Simple 1-D Carrier Model

Linear Full-Wave
Eigenvalue Simulations
(3D-VFEM)

Nonlinear Optical Response
Both CW and Pulsed Operation

[*Bravo-Abad, JLT 25, 2539, 2007*]
[*Wang, JLT 31, 313, 2013*]

Perturbation Theory for instantaneous nonlinear effects

Uncoupled nonlinear resonator

- ❑ *Linear regime*: Unperturbed resonant frequency ω_0
- ❑ *Nonlinear regime*:
  - **Frequency shift Re{Δ*ω*}** due to **nonlinear self-action**
  - **Nonlinear losses Im{Δ*ω*}** due to **TPA**

$$\Delta\omega = \left(\gamma_{\mathrm{Kerr}} + j\gamma_{\mathrm{TPA}}\right) W \begin{cases} \gamma_{\mathrm{Kerr}} = \dfrac{1}{4}\left(\dfrac{\omega_0}{c_0}\right)^3 c_0 \omega_0 \kappa_{\mathrm{Kerr}} n_2^{\max} \\ \\ \gamma_{\mathrm{TPA}} = \dfrac{1}{8}\left(\dfrac{\omega_0}{c_0}\right)^3 c_0^2 \kappa_{\mathrm{TPA}} \beta_{\mathrm{TPA}}^{\max} \end{cases}$$

$$\kappa_{\mathrm{Kerr}} = \left(\frac{c_0}{\omega_0}\right)^3 \frac{\dfrac{1}{3}\iiint_V n_2(\mathbf{r}) n^2(\mathbf{r}) \left[\left|\mathbf{E}_0 \cdot \mathbf{E}_0\right|^2 + 2\left|\mathbf{E}_0\right|^4\right] \mathrm{d}V}{\left[\dfrac{1}{2}\iiint_V n^2(\mathbf{r})\left|\mathbf{E}_0\right|^2 \mathrm{d}V\right]^2 n_2^{\max}}$$

$$\kappa_{\mathrm{TPA}} = \left(\frac{c_0}{\omega_0}\right)^3 \frac{\dfrac{1}{3}\iiint_V \beta_{\mathrm{TPA}}(\mathbf{r}) n^2(\mathbf{r}) \left[\left|\mathbf{E}_0 \cdot \mathbf{E}_0\right|^2 + 2\left|\mathbf{E}_0\right|^4\right] \mathrm{d}V}{\left[\dfrac{1}{2}\iiint_V n^2(\mathbf{r})\left|\mathbf{E}_0\right|^2 \mathrm{d}V\right]^2 \beta_{\mathrm{TPA}}^{\max}}$$

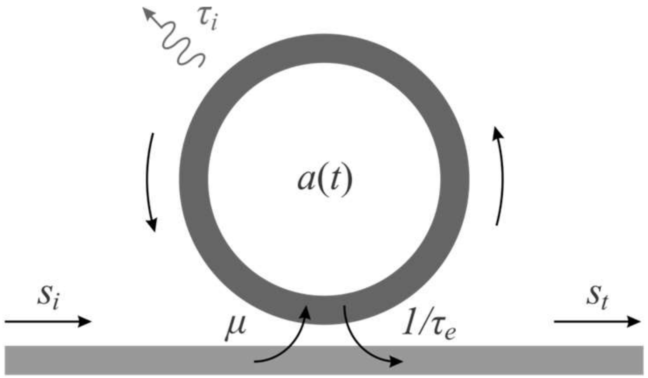
- Both proportional to stored energy *W*
- Both proportional to **nonlinear feedback parameters** *κ*, measuring overlap w/ DDMEBT and Si, respectively
- Obtained from **linear** full-wave simulation (3D-VFEM)

Temporal coupled mode theory (CMT)

$$\frac{\mathrm{d}a}{\mathrm{d}t} = j\left(\omega_0 + \mathrm{Re}\{\Delta\omega\}\right)a - \frac{1}{\tau_i}a - \frac{1}{\tau_e}a - \frac{1}{\tau_{\mathrm{TPA}}}a + \mu s_i$$

$$s_t = s_i + \mu a$$

$$\mathrm{d}P_{\mathrm{TPA}} = \frac{1}{2}\mathrm{Re}\left\{\mathbf{E}_0^* \cdot j\omega\mathbf{P}^{(3)}\right\}\mathrm{d}V \Rightarrow P_{\mathrm{TPA}} = 2\gamma_{\mathrm{TPA}}\left|a\right|^4$$



$a(t)$	cavity amplitude, $\lvert a(t)\rvert^2=W$
ω_0	unperturbed resonant frequency
Re{$\Delta\omega$}	nonlinear frequency shift
τ	photon lifetime, $\tau=2Q/\omega$
$1/\tau_{\mathrm{TPA}}$	TPA losses factor
μ	coupling coefficient, $\mu=(2/\tau_e)^{1/2}$
s	w/g mode amplitudes, $\lvert s\rvert^2=P$

Normalized detuning: $\delta = \tau_i(\omega - \omega_0)$

Quality factor ratio: $r_Q = Q_i \,/\, Q_e = \tau_i \,/\, \tau_e$

TPA losses factor: $r_{\mathrm{TPA}} = \dfrac{\gamma_{\mathrm{TPA}}}{\gamma_{\mathrm{Kerr}}} = \dfrac{\kappa_{\mathrm{TPA}}}{\kappa_{\mathrm{Kerr}}}\dfrac{\beta_{\mathrm{TPA}}^{\max}}{2k_0 n_2^{\max}}$

Steady-state response

$$\frac{p_{\mathrm{out}}}{p_{\mathrm{in}}} = \frac{(\delta + p_i)^2 + (1 - r_Q + r_{\mathrm{TPA}}p_i)^2}{(\delta + p_i)^2 + (1 + r_Q + r_{\mathrm{TPA}}p_i)^2}$$

$$p_{\mathrm{TPA}} = r_{\mathrm{TPA}}p_i^2$$

with

$$p_i = p_{\mathrm{in}} - p_{\mathrm{out}} - p_{\mathrm{TPA}}$$

Kerr characteristic power

$$P_0^{\mathrm{Kerr}} = \frac{2}{\tau_i^2\gamma_{\mathrm{Kerr}}} = \frac{2}{\left(\dfrac{\omega_0}{c_0}\right)^2 \kappa_{\mathrm{Kerr}}Q_i^2 n_2^{\max}} \propto \frac{1}{\kappa_{\mathrm{Kerr}}Q_i^2}$$

[*Tsilipakos, Christopoulos and Kriezis, JLT 34, 1333, 2016*]

Temporal coupled mode theory (CMT)

Steady-state response

$$\frac{p_{\text{out}}}{p_{\text{in}}} = \frac{(\delta + p_i)^2 + (1 - r_Q + r_{\text{TPA}} p_i)^2}{(\delta + p_i)^2 + (1 + r_Q + r_{\text{TPA}} p_i)^2}$$

$$p_{\text{TPA}} = r_{\text{TPA}} p_i^2$$

with

$$p_i = p_{\text{in}} - p_{\text{out}} - p_{\text{TPA}}$$

- **Closed-form** 2x2 polynomial system
- Allows for constructing the hysteresis loop
- Admits **three** real positive solutions (for appropriate p_{in} levels and detuning)
- **Detuning threshold:**
  - $\partial p_{\text{in}} / \partial p_{\text{out}} = 0$ and positive discriminant

$$\delta < -\frac{(1 + r_Q)\left(\sqrt{3} + r_{\text{TPA}}\right)}{1 - \sqrt{3} r_{\text{TPA}}}$$

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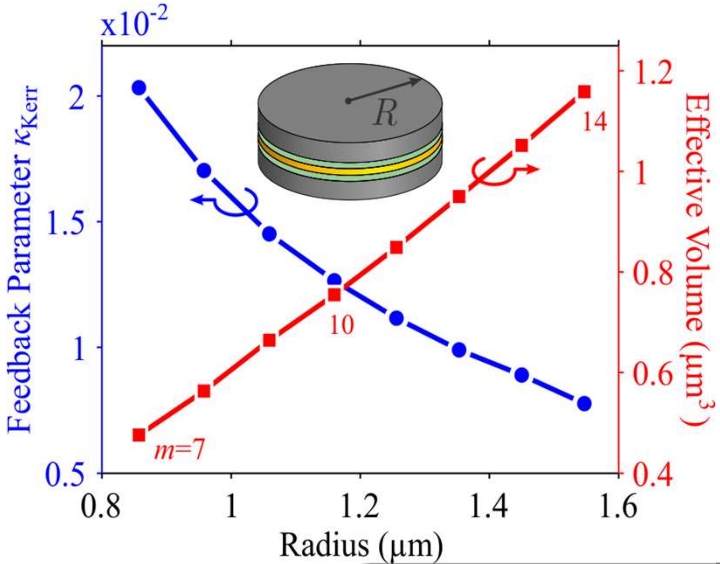
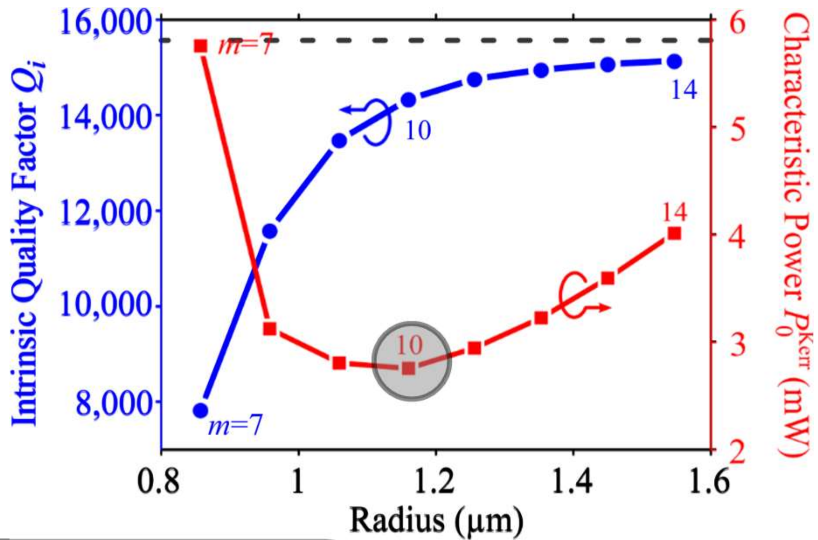
The uncoupled disk as an eigenvalue problem: P_0 minimization

Parametric analysis w.r.t. radius *R* for air-suspended disk

- R < 0.8 μm → Significant radiation losses
- R > 1.5 μm → Q_i bound by resistive losses (~15,500)
- κ, Q_i : **Opposing trends** with radius
- **Minimum P_0** or maximum κQ_i^2 product





Optimum value: R = 1.16 μm

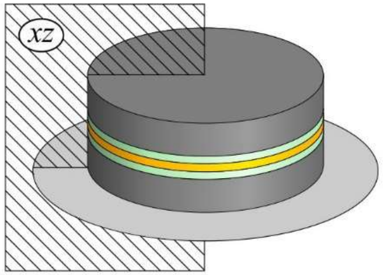
- $\kappa_{\mathrm{Kerr}} = 1.27\times10^{-2}$ | $\kappa_{\mathrm{TPA}} = 8.37\times10^{-4}$
- Q_i = 14330
- **P_0^{Kerr} = 2.75 mW**

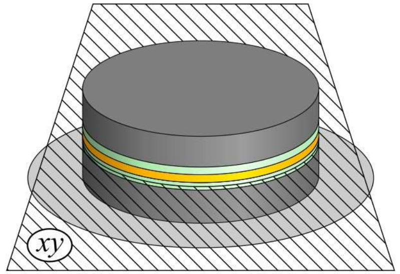
The uncoupled disk as an eigenvalue problem: Re-Symmetrization

- ❑ Mode de-symmetrizes
- ❑ Re-symmetrization process

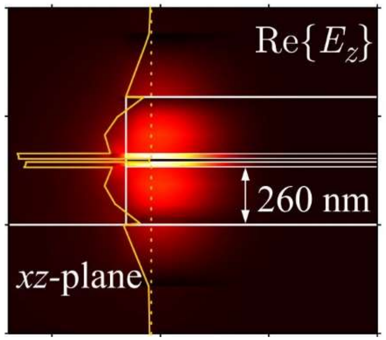
Resonant mode

(R = 1.158 μm, m = 10, λ_{res} = 1550 nm)

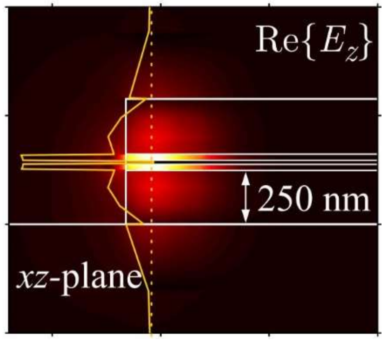


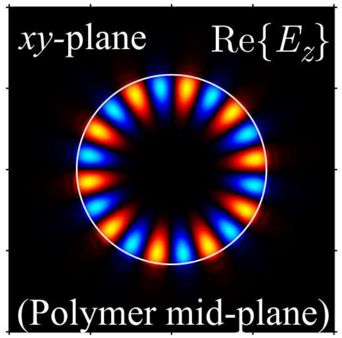


Asymmetric profile



Re-symmetrized profile





- ☑ Resistive losses restoration
- ☑ Similar κ
- ☒ Higher radiation losses (n_{SiO2}>n_{Air})
- ☑ **Low P_0^{Kerr}**

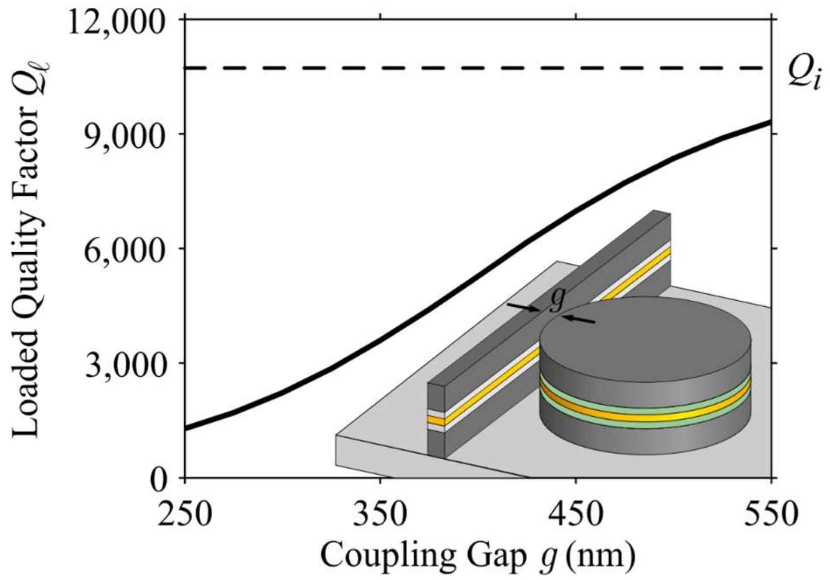
Optimum value: R = 1.158 μm

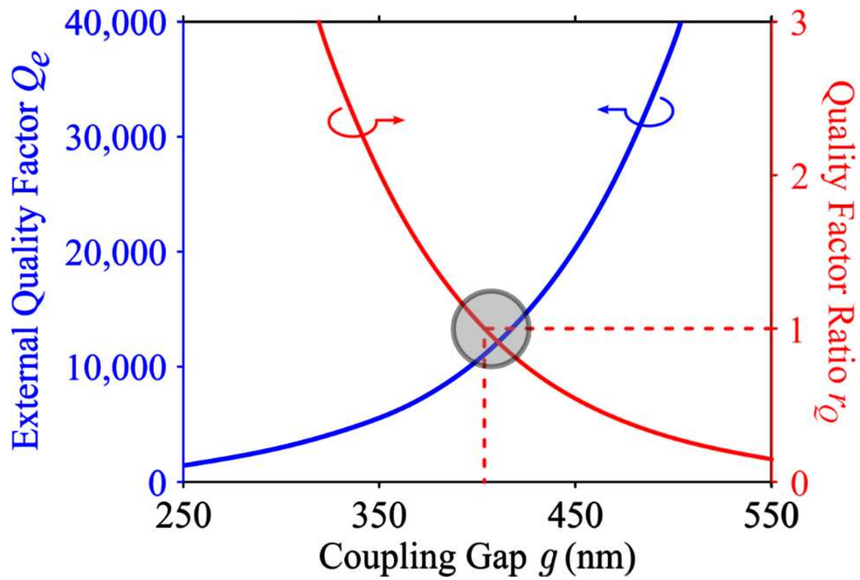
- ❑ κ_{Kerr} = 1.26×10^{-2} | κ_{TPA} = 8.3×10^{-4}
- ❑ Q_i = 10720
- ❑ **P_0^{Kerr} = 5 mW**

The coupled disk as an eigenvalue problem: Critical coupling

Parametric analysis w.r.t. coupling gap *g*

- **Loaded** quality factor Q_l
- **External** quality factor: $Q_e^{-1} = Q_l^{-1} - Q_i^{-1}$
- Quality factor **ratio** $r_Q = Q_i / Q_e$
- Critical coupling ($r_Q = 1$) for minimum transmission on resonance





Critical coupling (r_Q=1) → ***g* = 400 nm**

To recapitulate…

Physical system design

- min{P_0} → $\boldsymbol{R}$ **= 1.158 μm**
- $r_Q = 1$ → $\boldsymbol{g}$ **= 400 nm**

System Parameters

- $Q_i = 10720$
- $\kappa_{\mathrm{Kerr}} = 1.27\times10^{-2}$ → $P_0^{\mathrm{Kerr}} = 5$ mW
- $\kappa_{\mathrm{TPA}} = 8.3\times10^{-4}$ → $r_{\mathrm{TPA}} = 0.0095$
- $\delta_{\mathrm{th}} = -3.54$
- $\delta = 1.57\delta_{\mathrm{th}} = -5.55$ → $\lambda_0 = 1550.4$ nm

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Perturbation Theory for FCEs

- **Frequency shift $\Delta\omega_{\mathrm{FCD}}$** due to **Free Carrier Dispersion**
- **Nonlinear losses $1/\tau_{\mathrm{FCA}}$** due to **Free Carrier Absorption**

$$\frac{\Delta\omega_{\mathrm{FCE}}}{\omega_0} = \frac{\Delta\omega_{\mathrm{FCD}} + j\tau_{\mathrm{FCA}}^{-1}}{\omega_0} = -\frac{1}{2}\frac{\iiint_V \Delta\chi_{\mathrm{FCE}}^{(1)} \left|\mathbf{E}_0\right|^2 \mathrm{d}V}{\iiint_V n^2 \left|\mathbf{E}_0\right|^2 \mathrm{d}V}$$

$$\begin{aligned} \sigma_n^e &= 8.8\times 10^{-28}\,\mathrm{m}^3 \\ \sigma_n^h &= 4.6\times 10^{-28}\,\mathrm{m}^3 \\ \sigma_a &= 14.5\times 10^{-22}\,\mathrm{m}^2 \end{aligned}$$

$$\Delta\chi_{\mathrm{FCE}}^{(1)} = 2n\Delta n_{\mathrm{FCD}} - j\frac{nc_0}{\omega}\Delta a_{\mathrm{FCA}} \begin{cases} \Delta n_{\mathrm{FCD}} = -\sigma_n^e N - \left(\sigma_n^h N\right)^{0.8} = -\left(\sigma_n^e + \left(\sigma_n^h\right)^{0.8}\bar{N}^{-0.2}\right)N = -\sigma_n\left(\bar{N}\right)N \\ \Delta a_{\mathrm{FCA}} = -\sigma_a N \end{cases}$$

Free Carrier Density

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\frac{N}{\tau_c} + \frac{1}{2\hbar\omega_0}\frac{\mathrm{d}P_{\mathrm{TPA}}}{\mathrm{d}V}$$

Weighted Free Carrier Density

$$\bar{N} = \frac{\iiint_V N(\mathbf{r})\left|\mathbf{E}_0(\mathbf{r})\right|^2 \mathrm{d}V}{\iiint_V \left|\mathbf{E}_0(\mathbf{r})\right|^2 \mathrm{d}V}$$

Coordinate-independent Free Carrier Density

$$\boxed{\frac{\mathrm{d}\bar{N}}{\mathrm{d}t} = -\frac{\bar{N}}{\tau_c} + \gamma_{\mathrm{N}}W^2}$$

- Carrier lifetime τ_c ~ 0.5 ns
- Reduction to τ_c ~ 10 ps by carrier sweeping or proton bombardment

Perturbation Theory for FCEs

Continuous Wave Conditions

$$\Delta\omega_{\mathrm{FCE}} = \underbrace{\left(\gamma_{\mathrm{FCD}} + j\gamma_{\mathrm{FCA}}\right)}W^2$$

$$\gamma_{\mathrm{FCD}} = \frac{1}{16}\left(\frac{\omega_0}{c_0}\right)^6 \frac{\sigma_n\left(\bar{N}\right)\tau_c c_0^2}{\hbar}\kappa_{\mathrm{FCE}}\beta_{\mathrm{TPA}}^{\max}$$

$$\gamma_{\mathrm{FCA}} = \frac{1}{32}\left(\frac{\omega_0}{c_0}\right)^6 \frac{\sigma_a\tau_c c_0^3}{\hbar\omega_0}\kappa_{\mathrm{FCE}}\beta_{\mathrm{TPA}}^{\max}$$

$$\kappa_{\mathrm{FCE}} = \left(\frac{c_0}{\omega_0}\right)^6 \frac{\frac{1}{3}\iiint_V \beta_{\mathrm{TPA}}(\mathbf{r})n^3(\mathbf{r})\left[\left|\mathbf{E}_0\cdot\mathbf{E}_0\right|^2 + 2\left|\mathbf{E}_0\right|^4\right]\left|\mathbf{E}_0\right|^2 \mathrm{d}V}{\left[\frac{1}{2}\iiint_V n^2(\mathbf{r})\left|\mathbf{E}_0\right|^2 \mathrm{d}V\right]^3 \beta_{\mathrm{TPA}}^{\max}}$$

$$\kappa_{\mathrm{N}} = \left(\frac{c_0}{\omega_0}\right)^6 \frac{\frac{1}{3}\iiint_V \beta_{\mathrm{TPA}}(\mathbf{r})n^3(\mathbf{r})\left[\left|\mathbf{E}_0\cdot\mathbf{E}_0\right|^2 + 2\left|\mathbf{E}_0\right|^4\right]\left|\mathbf{E}_0\right|^2 \mathrm{d}V}{\iiint_V \left|\mathbf{E}_0\right|^2 \mathrm{d}V\left[\frac{1}{2}\iiint_V n^2(\mathbf{r})\left|\mathbf{E}_0\right|^2 \mathrm{d}V\right]^2 \beta_{\mathrm{TPA}}^{\max}}$$

Pulsed Conditions

$$\Delta\omega_{\mathrm{FCE}} = \underbrace{\left(\gamma_{\mathrm{FCD}}^{\mathrm{dyn}}\sigma_n\left(\bar{N}\right) + j\gamma_{\mathrm{FCA}}^{\mathrm{dyn}}\right)}\bar{N}$$

$$\gamma_{\mathrm{FCD}}^{\mathrm{dyn}} = \frac{1}{2}\omega_0\frac{\kappa_{\mathrm{FCE}}}{\kappa_{\mathrm{N}}}$$

$$\gamma_{\mathrm{FCA}}^{\mathrm{dyn}} = \frac{1}{4}c_0\frac{\kappa_{\mathrm{FCE}}}{\kappa_{\mathrm{N}}}\sigma_a$$

$$\gamma_{\mathrm{N}} = \frac{1}{8}\left(\frac{\omega_0}{c_0}\right)^6 \frac{c_0^2}{\hbar\omega_0}\kappa_{\mathrm{N}}\beta_{\mathrm{TPA}}^{\max}$$

- Both proportional to W^2 or N
- Nonlinear parameters κ measuring overlap w/ Si
- Obtained from **linear** full-wave simulation (3D-VFEM)
- **Different** from κ_{TPA}

CMT with FCEs

CMT equations

$$\frac{\mathrm{d}a}{\mathrm{d}t} = j\left(\omega_0 + \Delta\omega_{\mathrm{Kerr}} + \Delta\omega_{\mathrm{FCD}}\right)a - \frac{1}{\tau_i}a - \frac{1}{\tau_e}a - \frac{1}{\tau_{\mathrm{TPA}}}a - \frac{1}{\tau_{\mathrm{FCA}}}a + \mu s_i$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\frac{N}{\tau_c} + \frac{1}{2\hbar\omega_0}\frac{\mathrm{d}P_{\mathrm{TPA}}}{\mathrm{d}V}$$

[*Tsilipakos, Christopoulos and Kriezis, JLT 34, 1333, 2016*]

CW Normalization Process

$$\tilde{u} = \sqrt{\tau_i \gamma_{\mathrm{Kerr}}}\,\tilde{a} \qquad \tilde{\psi}_i = \tilde{s}_i \,/\, \sqrt{P_0^{\mathrm{Kerr}}}$$

$$\boxed{j2\sqrt{r_Q}\tilde{\psi}_i = j\left(\delta + \left|\tilde{u}\right|^2 - r_{\mathrm{FCD}}\left|\tilde{u}\right|^4\right)\tilde{u} + \left(1 + r_Q + r_{\mathrm{TPA}}\left|\tilde{u}\right|^2 + r_{\mathrm{FCA}}\left|\tilde{u}\right|^4\right)\tilde{u}}$$

- ☑ Can be solved numerically for producing bistability loop
- ☒ Not in polynomial form, cannot give detuning limit

$$r_{\mathrm{FCD}} = \frac{\gamma_{\mathrm{FCD}}}{\tau_i \gamma_{\mathrm{Kerr}}^2} = \frac{\sigma_n\left(\bar{N}\right)\tau_c}{\tau_i \omega_0^2 \hbar}\frac{\kappa_{\mathrm{FCE}}}{\kappa_{\mathrm{Kerr}}^2}\frac{\beta_{\mathrm{TPA}}^{\max}}{\left(n_2^{\max}\right)^2} \qquad r_{\mathrm{FCD}}^{\mathrm{dyn}}\left(\bar{n}\right) = \frac{r_{\mathrm{FCD}}}{\tau_c'}$$

$$r_{\mathrm{FCA}} = \frac{\gamma_{\mathrm{FCA}}}{\tau_i \gamma_{\mathrm{Kerr}}^2} = \frac{\sigma_a \tau_c c_0}{2\tau_i \omega_0^3 \hbar}\frac{\kappa_{\mathrm{FCE}}}{\kappa_{\mathrm{Kerr}}^2}\frac{\beta_{\mathrm{TPA}}^{\max}}{\left(n_2^{\max}\right)^2} \qquad r_{\mathrm{FCA}}^{\mathrm{dyn}} = \frac{r_{\mathrm{FCA}}}{\tau_c'}$$

Pulsed Normalization Process

$$\bar{n} = \left(\tau_i \gamma_{\mathrm{Kerr}}^2 \,/\, \gamma_{\mathrm{N}}\right)\bar{N} \qquad \tau_c' = \tau_c \,/\, \tau_i$$

$$\boxed{\begin{aligned} \frac{\mathrm{d}\tilde{u}}{\mathrm{d}t} &= j\left(-\delta - \left|\tilde{u}\right|^2 + r_{\mathrm{FCD}}^{\mathrm{dyn}}\left(\bar{n}\right)\bar{n}\right)\tilde{u} - \left(1 + r_Q + r_{\mathrm{TPA}}\left|\tilde{u}\right|^2 + r_{\mathrm{FCA}}^{\mathrm{dyn}}\bar{n}\right)\tilde{u} + j2\sqrt{r_Q}\tilde{\psi}_i \\ \frac{\mathrm{d}\bar{n}}{\mathrm{d}t} &= -\frac{\bar{n}}{\tau_c} + \left|\tilde{u}\right|^4 \end{aligned}}$$

- ☑ Can be also solved numerically
- ☑ Includes the dependence of *n* on $\sigma_n(n)$
- ☑ Allows for calculating device response under ultrafast pulses

Summing up nonlinear parameters

Nonlinear Parameters

Parameter	Value
r_{TPA}	0.0095
$r_{\text{FCD}}\left(\tau_c = 8\ \text{ps}\right)$	$0.0294\left(P_{\text{in}} = 40\ \text{mW}\right)$
$r_{\text{FCA}}(\tau_c = 8\ \text{ps})$	0.0019
$r_{\text{FCD}}^{\text{dyn}}$	$0.0206 + 0.0972\overline{n}^{-0.2}$
$r_{\text{FCA}}^{\text{dyn}}$	0.0042

Depends on carrier lifetime τ_c

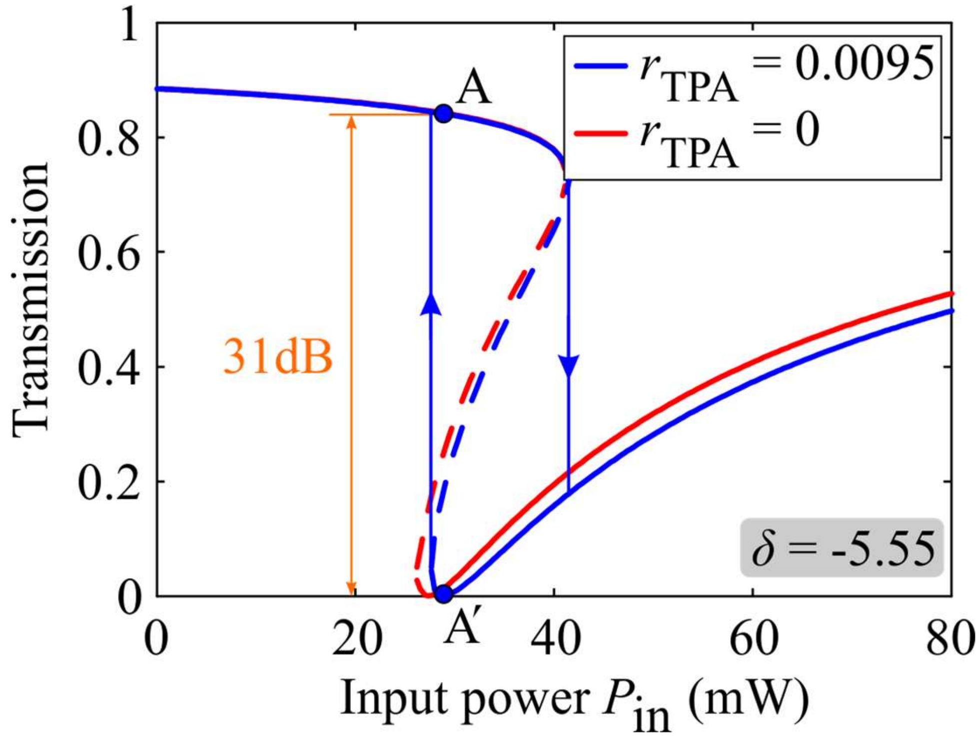
Depends on input power P_{in} via dependence on N

Depends on normalized carrier density n

Presentation outline

- **Nonlinear Long-Range Hybrid Plasmonic Travelling-Wave Disk Resonator**
  - Long-Range Hybrid Plasmonic Waveguide
  - Physical System: Side-Coupled Disk Resonator
  - Perturbation Theory & CMT Framework: Kerr effect and Two Photon Absorption
  - System Design
  - Perturbation Theory & CMT Framework: Free Carrier Effects
  - CW Performance Assessment
  - Temporal Response
  - Stability Analysis and Self-Pulsation

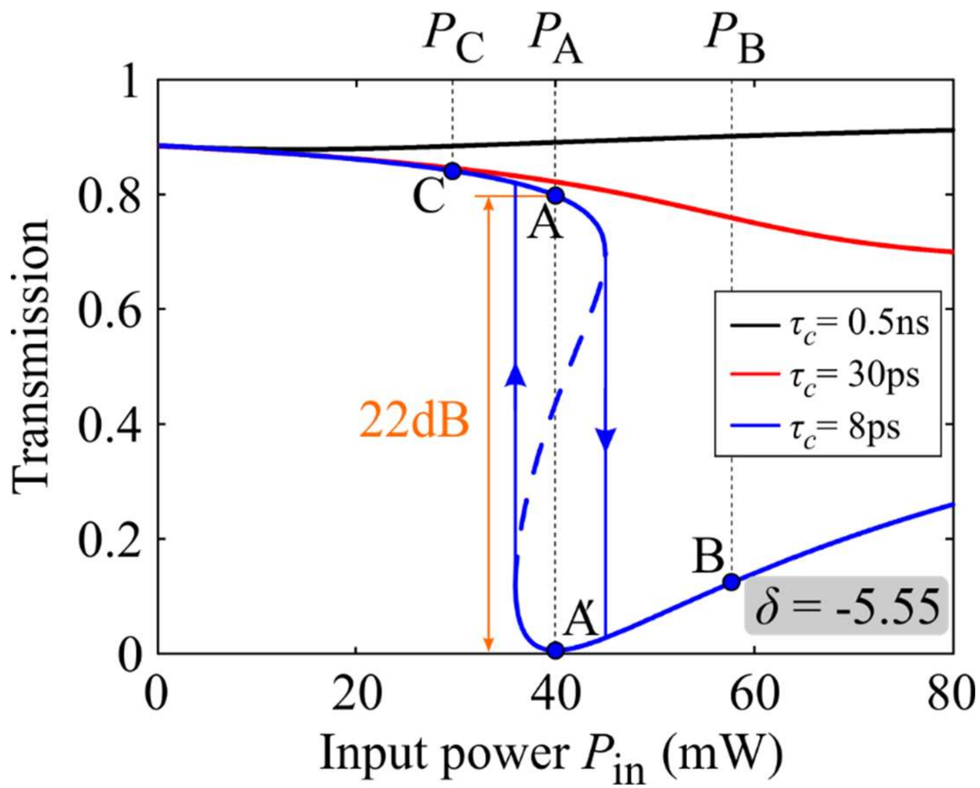
Bistability curve, TPA enclosure



Instantaneous Effects Performance

- ***P*$_{in}$ = 28 mW** for bistability
- IL = 0.75 dB @ point A
- **ER = 31 dB** @ $P_{A'}$
- Weak contribution of TPA to ER and P_{in} (red curve)

Bistability curve, TPA + FCEs enclosure



Full Nonlinear Performance

- τ_c < 10 ps for Kerr bistability → weak influence of FCEs
- $\boldsymbol{P_{in}}$ **= 40 mW** for bistability
- IL = 0.9 dB @ point A
- **ER = 22 dB** @ P_A
- Points B, C for toggling between bistable states A, A′

Presentation outline

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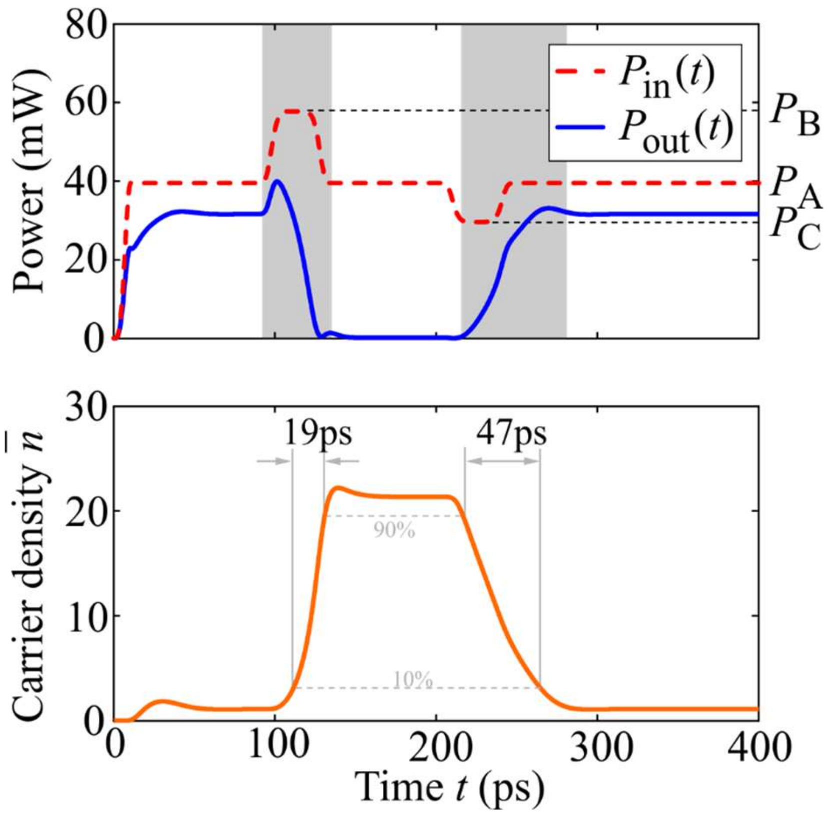
Temporal response

Memory Operation

- Initially $P_{in} = P_A$ → System at **high-output** state
- 3rd-order **super-Gaussian pulses** (FWHM = 26 ps) for **toggling states**
  - 1st pulse (peak) → *ABA′* → **low-output** state
  - 2nd pulse (dip) → *A′CA* → **high-output** state



Carrier evolution

- Rise time: 19 ps
  - Limited by cavity lifetime (FCs generated instantaneously)
- Fall time: 47 ps
  - Limited by cavity & carrier lifetime (FCs recombine with τ_c)
- **Ultrafast** memory operation despite FCEs (> 10 Gbps)

Presentation outline

- **Nonlinear Long-Range Hybrid Plasmonic Travelling-Wave Disk Resonator**
  - Long-Range Hybrid Plasmonic Waveguide
  - Physical System: Side-Coupled Disk Resonator
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  - System Design
  - Perturbation Theory & CMT Framework: Free Carrier Effects
  - CW Performance Assessment
  - Temporal Response
  - Stability Analysis and Self-Pulsation

Stability Analysis

CW Maps (δ - P_{in} plane)

1st order stability analysis

$$\tilde{u}' = \tilde{u} + \delta\tilde{u} \quad \overline{n}' = \overline{n} + \delta\overline{n}$$

After some algebra

$$\mathrm{d}\varepsilon \,/\, \mathrm{d}t = \mathbf{J}\varepsilon \text{ with } \varepsilon = \left[\delta\tilde{u} \quad \delta\tilde{u}^* \quad \delta\overline{n}\right]^T$$

Eigenvalues overview as in
[*Chen, Opt. Expr. 20, 7454, 2012*]

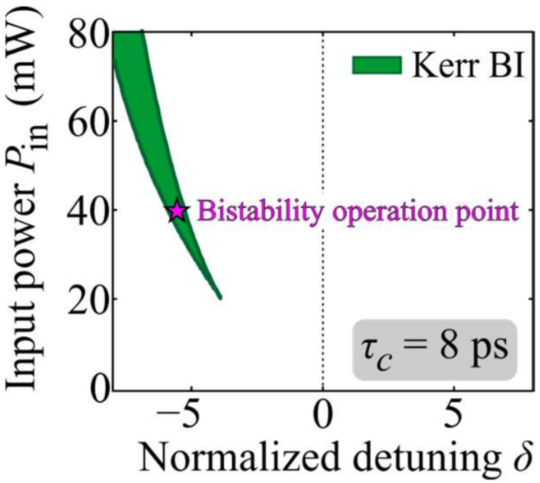
Self-Pulsation

- ❑ Oscillating output for CW input
- ❑ Appears when two phenomena have similar lifetimes
  - FCD lifetime (τ_c ~ 40 ps)
  - Coupling between bus waveguide and resonator ($\tau_e = \tau_i$ ~ 18 ps)

Eigenvalues Overview

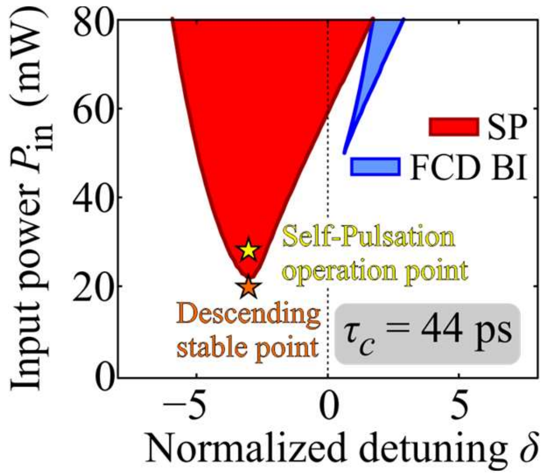
- ❑ **J**: 3x3 matric
- ❑ Three eigenvalues
  - λ_1 Real
  - $\lambda_{2,3}$ Complex
- ❑ Re{$\lambda_{1,2,3}$} < 0 → **Stable**
- ❑ Re{λ_1} > 0 && Re{$\lambda_{2,3}$} < 0 → **Bistability (BI)**
- ❑ Re{λ_1} < 0 && Re{$\lambda_{2,3}$} > 0 → **Self-Pulsation (SP)**
- ❑ Re{λ_1} > 0 && Re{$\lambda_{2,3}$} > 0 → **BI + SP**

Self-Pulsation: CW maps



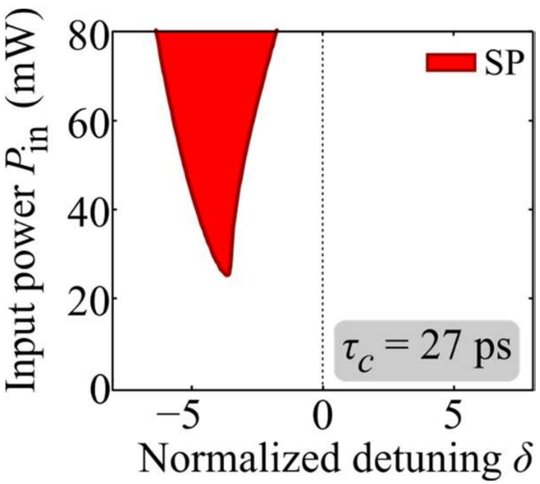
SP
- No SP region

Kerr-induced BI
- $P_0^{Kerr} >> P_0^{FCD}$
- Negative detunings



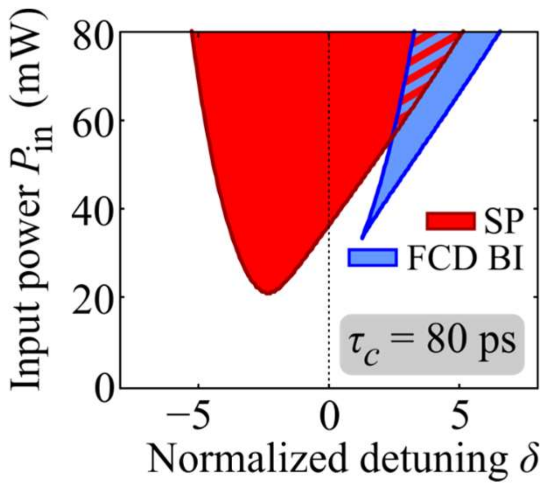
SP
- Both negative and positive detunings

FCD-induced BI
- $P_0^{FCD} > P_0^{Kerr}$
- Positive detunings



SP
- Negative detunings

BI
- $P_0^{Kerr} \sim P_0^{FCD}$
- No BI region

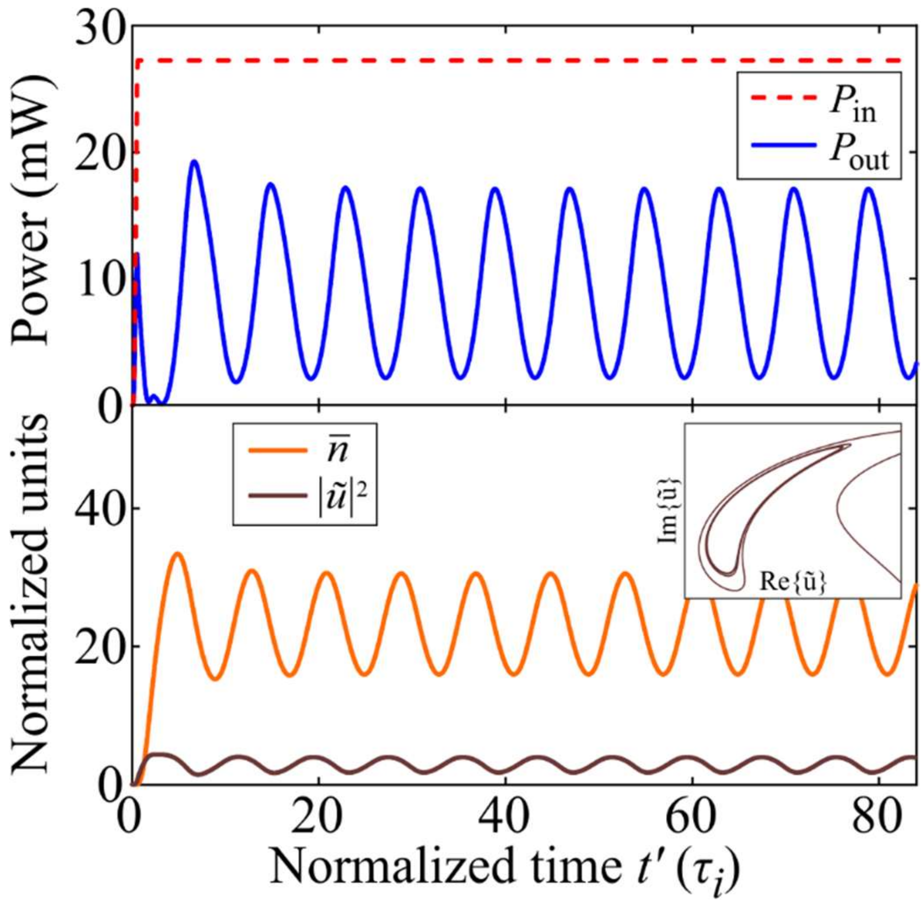


SP
- Both negative and positive detunings

FCD-induced BI
- $P_0^{FCD} >> P_0^{Kerr}$
- Positive detunings
- Overlapping area

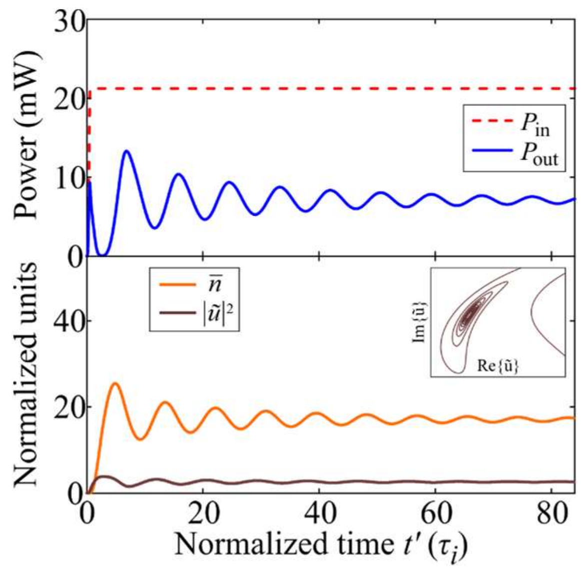
Self-Pulsation: Temporal Response

SP temporal response



- CW input power (27 mW)
- Oscillating output
  - Almost sinusoidal (Non-circular phase-space diagram)
  - *f* = 7.5 GHz
  - High modulation depth (>0.8)
  - *f* can be tuned through P_{in}
- Tunable integrated clock application **!**

Descending Oscillations (stable state)



Conclusion

❑ Summary

- **Practical** plasmonic component for Bistability and Self-Pulsation
- **TPA** and **FCEs** encapsulation
- ❑ Kerr-induced bistability
  - **Optical memory** operation
  - **Ultrafast** response
- ❑ Self-Pulsation
  - Full **optical clock** implementation
  - **Tunable** output frequency

❑ To probe further ...

- FCD bistability
- Thermal bistability
  - TPA, FCA and Joule heating
- Graphene comprising bistability
- Excitability

Thank you!

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And… there is more

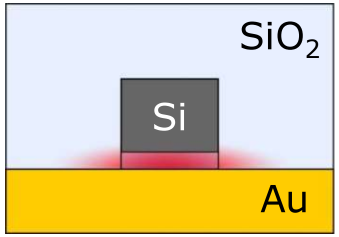
Back up material !

Scope

❑ Nonlinear control in guided-wave plasmonics

☑ Sub-λ confinement
☒ Resistive losses

→ *Hybrid plasmonic waveguides* (best compromise)



[*Oulton, Nat. Photon. 2, 2008*]
[*Wu, Opt. Express 18, 2010*]

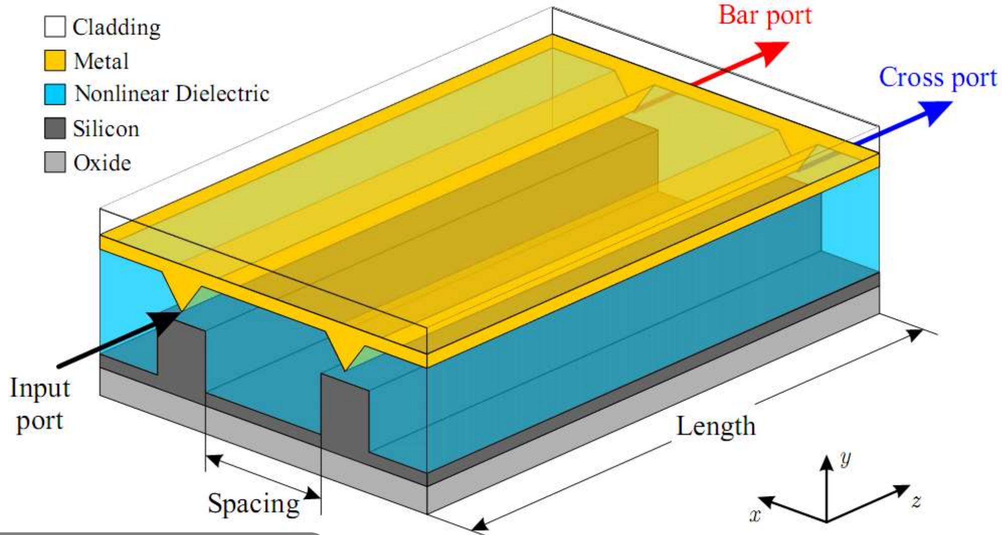
❑ Nonlinear phenomena

- Instantaneous: Kerr effect and Two Photon Absorption
- Non-instantaneous: Free Carrier Effects

❑ Nonlinear directional coupler

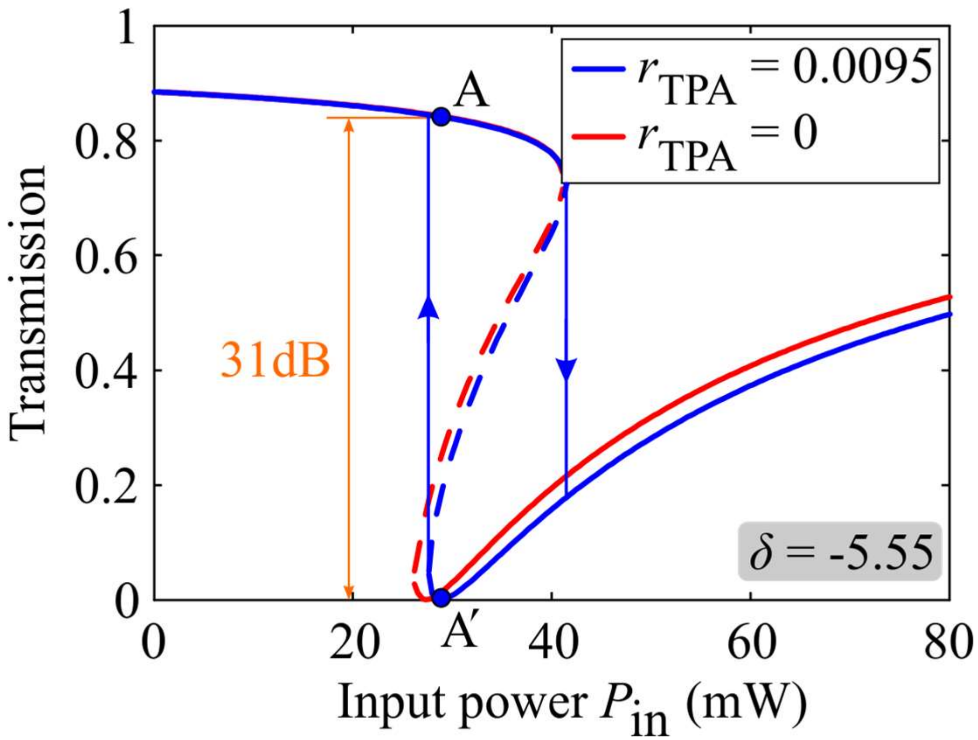
☒ P_{th} > 10s or even 100s W **!**
(loss → short interaction length)

[*Milián, APL 98, 2011*]
[*Kriesch, CLEO/QELS 2012*]
[*Pitilakis, JOSA B 30, 2013*]



Resonator enhanced… → Optical bistability

Bistability curve, TPA enclosure



ER compensation

- New critical coupling condition

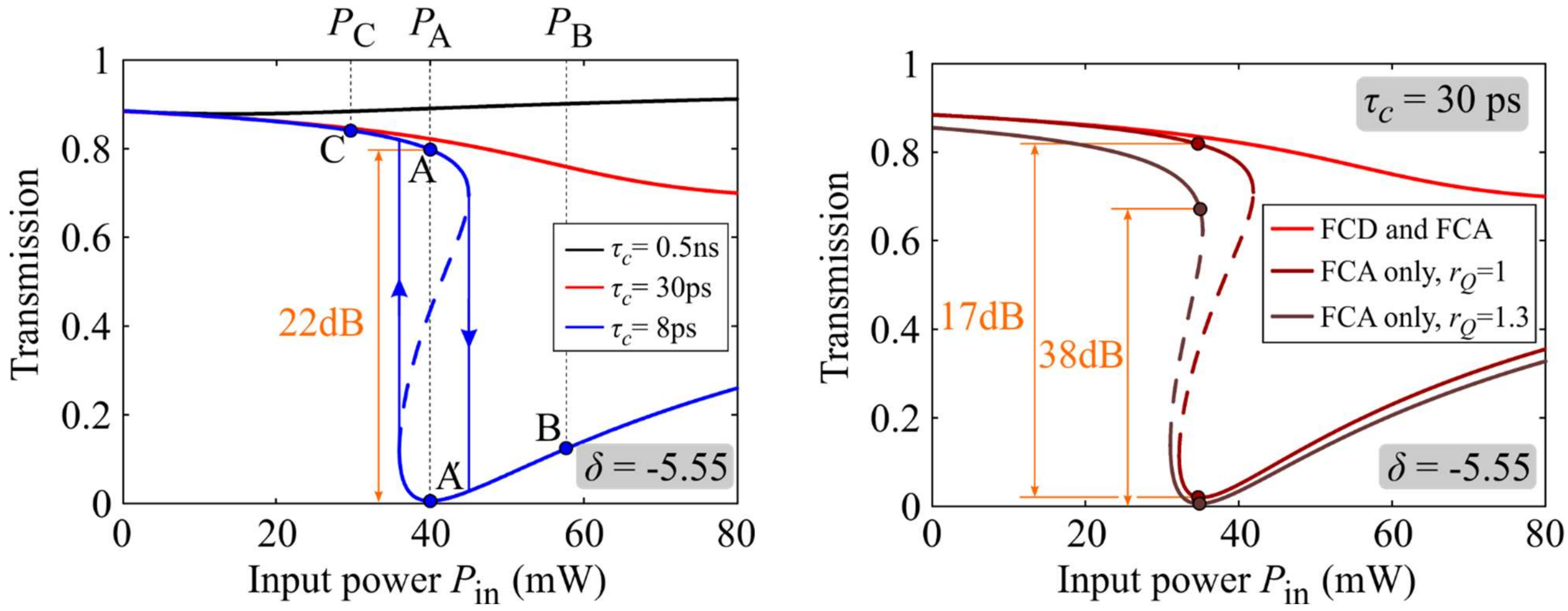
$$Q_e^{-1} = Q_i^{-1} + Q_{\text{TPA}}^{-1}$$

- Modify r_Q ($r'_Q > r_Q$)
- Q_{TPA} depends on P_{in} → compensation only for a certain point
- **Not necessary with TPA**

Performance

- **P_{in} = 28 mW** for bistability
- IL = 0.75 dB @ point A
- **ER = 31 dB** @ $P_{A'}$
- Weak contribution of TPA to ER and P_{in} (red curve)

Bistability curve, TPA + FCEs enclosure



Performance

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- $\boldsymbol{P_{in}}$ **= 40 mW** for bistability
- IL = 0.9 dB @ point A
- **ER = 22 dB** @ $P_{A'}$
- Points B, C for toggling between bistable states A, A′

FCE influence investigation

- FCD is more restrictive than FCA
- FCA losses can also be compensated
- r_Q = 1 → ER = 17 dB
- r'_Q = 1.3 → ER′ = 38 dB
- Bistability span and IL cannot be compensated

Self-Pulsation: CW maps

CW Maps (δ - P_{in} plane)

1^{st} order stability analysis

$$\tilde{u}' = \tilde{u} + \delta\tilde{u} \quad \overline{n}' = \overline{n} + \delta\overline{n}$$

After some algebra

$$\mathrm{d}\varepsilon \,/\, \mathrm{d}t = \mathbf{J}\varepsilon \text{ with } \varepsilon = \left[\delta\tilde{u} \quad \delta\tilde{u}^* \quad \delta\overline{n}\right]^T$$

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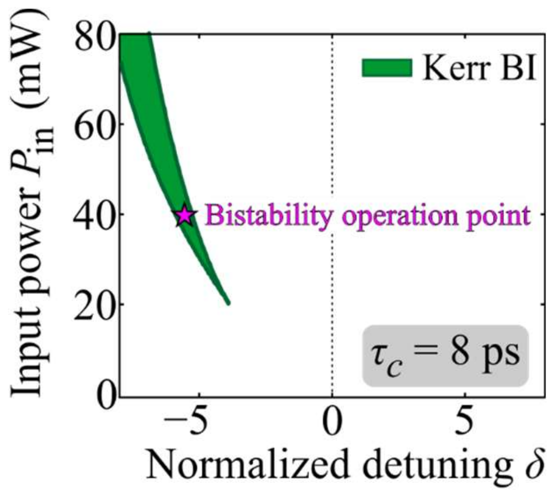
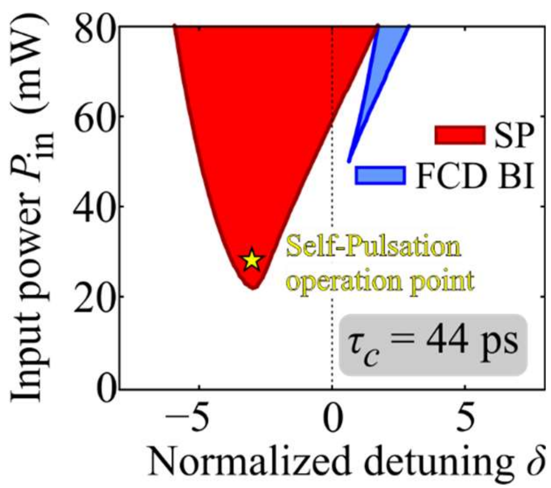


Kerr-induced BI

- $P_0^{Kerr} < P_0^{FCD}$
- Negative detunings
- Star marks BI operation point

Self-Pulsation (SP)

- ❑ Oscillating output for CW input
- ❑ Appears when two phenomena have similar lifetimes
  - FCD lifetime ($\tau_c \sim 40$ ps)
  - Coupling between bus waveguide and resonator ($\tau_e = \tau_i \sim 18$ ps)



SP

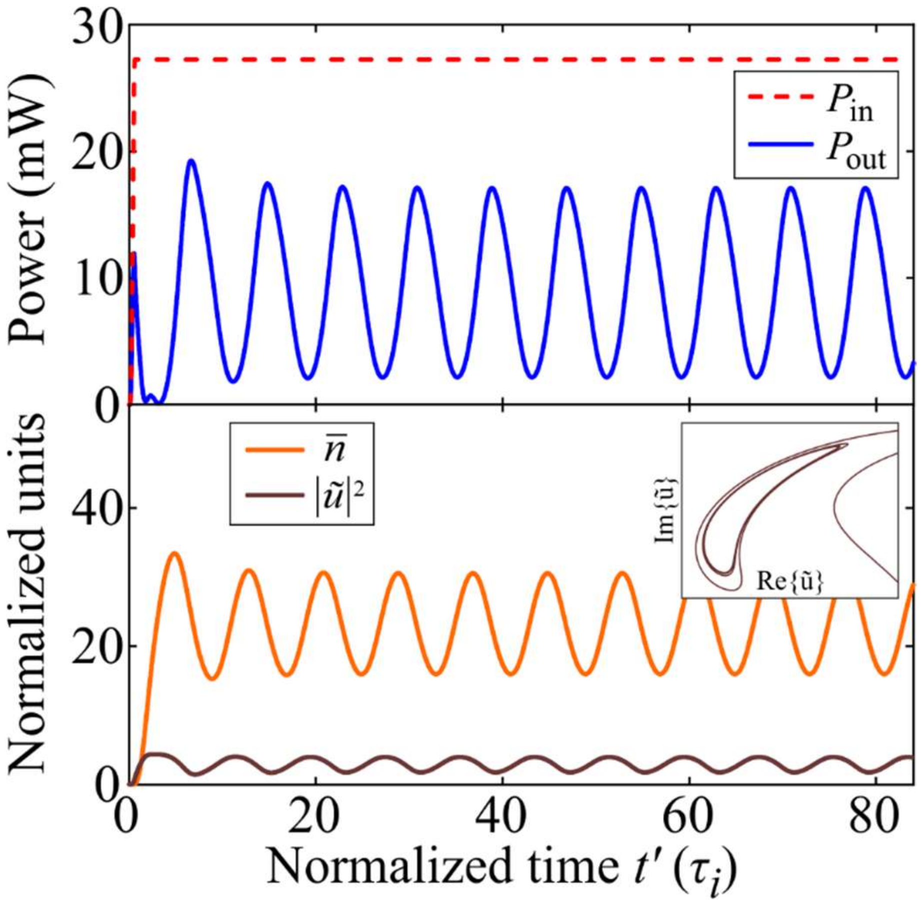
- Both negative and positive detunings
- $\tau_c \sim 2\tau_i$
- Star marks SP operation point

FCD-induced BI

- $P_0^{FCD} < P_0^{Kerr}$
- Positive detunings

Self-Pulsation: Temporal Response

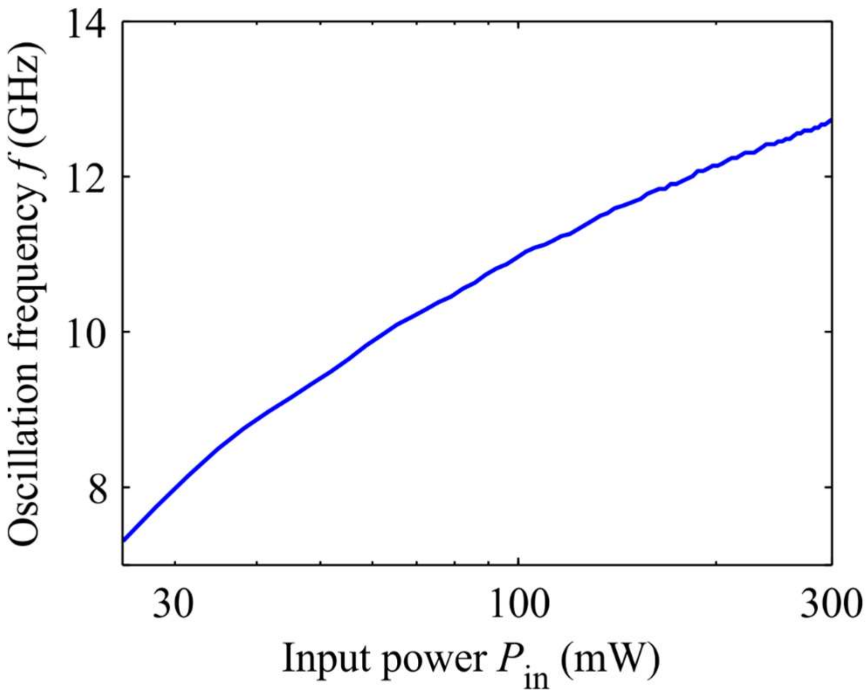
SP temporal response

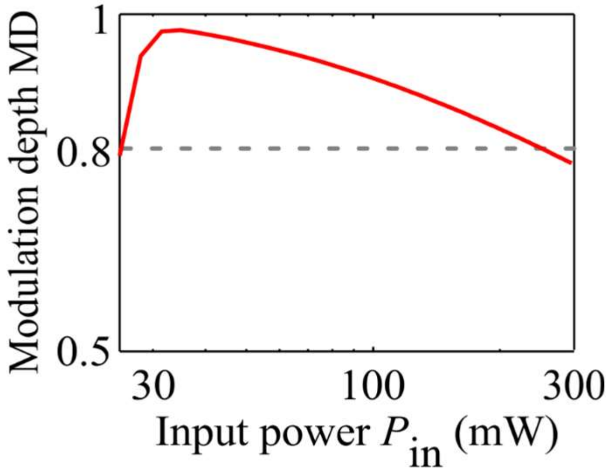


- ❑ CW input power (27 mW)
- ❑ Oscillating output
  - Carriers and cavity energy oscillates out of phase
  - Almost sinusoidal (Non-circular phase-space diagram)
  - f = 7.5 GHz
  - High modulation depth (>0.8)
  - f can be tuned through P_{in}
- ❑ Tunable integrated clock application **!**

Self-Pulsation: FM modulator

FM Modulator





- Amplitude Modulation at P_{in} → Frequency Modulation at f
  - Almost linear relation between f and P_{in}
  - High MD (>0.8) for $25 < P_{in} < 280$ mW
  - Assure sinusoidal response